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# Discrete exchange-springs in magnetic multilayer samples

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**Abstract.** The properties of soft magnetic exchange-springs in both bilayer and multilayer samples are investigated, with particular emphasis on the discrete nature of the spring. It is shown that, in a mean-field model, a very simple relationship exists between the bending field $B_B$, the exchange field $B_{EX}$, and the number of monolayers $N$ in the soft magnetic layer. For bilayers $B_B/B_{EX} = (\pi/2N)^2$, whereas for multilayers $B_B/B_{EX} = (\pi/N)^2$. In addition, it is shown that Jacobi elliptic functions, originally used by Goto *et al* for continuous bilayer springs, provide a surprisingly robust description of discrete bilayer and symmetric multilayer exchange-springs. Finally, the problem of soft exchange-spring penetration into neighbouring hard magnetic layers is discussed. Calculations show that this is an important effect, which leads to a reduction in the bending field $B_B$.

## 1. Introduction

In recent years, the properties of layered magnets, at the atomic level, have attracted much attention (Fullerton *et al* 1998, 1999, Coey and Skomski 1993, Skomski and Coey 1993). The underlying reason for this interest stems from the work of Coey and Skomski, who have argued, on theoretical grounds, that composite magnets with a giant energy product $(BH)_{MAX}$ of 120 MG Oe might be feasible, if the exchange-spring mechanism in nano-structured oriented magnets could be suppressed. This work has also been complemented with magnetic measurements on ferromagnetically coupled hard and soft multilayer samples, such as sputtered $SmCo_5$ and Fe layers. For reviews of the present status of this research field, and references contained therein, the reader is referred to Fullerton *et al* (1998, 1999).

From a theoretical point of view, a key paper, detailing the magnetic behaviour of a hard magnetic substrate coated with a soft Fe layer, was first given by Goto *et al* (1965). These authors showed that the angular dependence of a continuous 180° exchange-spring, in the soft magnetic layer of a bilayer sample, could be expressed in terms of Jacobi elliptic functions. In addition, they showed, both theoretically and experimentally, that the onset of the exchange-spring was characterized by a critical 'bending field $B_B$'. Moreover, this field is proportional to $1/w^2$, where $w$ is the thickness of the soft magnetic layer. In this paper, we extend their discussion to discrete bilayer and multilayer exchange-springs.

This paper is set out as follows. In the next section, a numerical method for calculating the explicit form of exchange-springs is described, as a function of applied magnetic field. In particular, computer calculations show that there is a simple relationship between the bending field $B_B$, the exchange field $B_{EX}$ and the number of monolayers $N$ in the soft magnetic layer $B_B/B_{EX} \propto 1/N^2$, for both bilayer and multilayer samples. In section 3, a fresh look at the analytical result of Goto *et al* (1965) is presented and discussed, but

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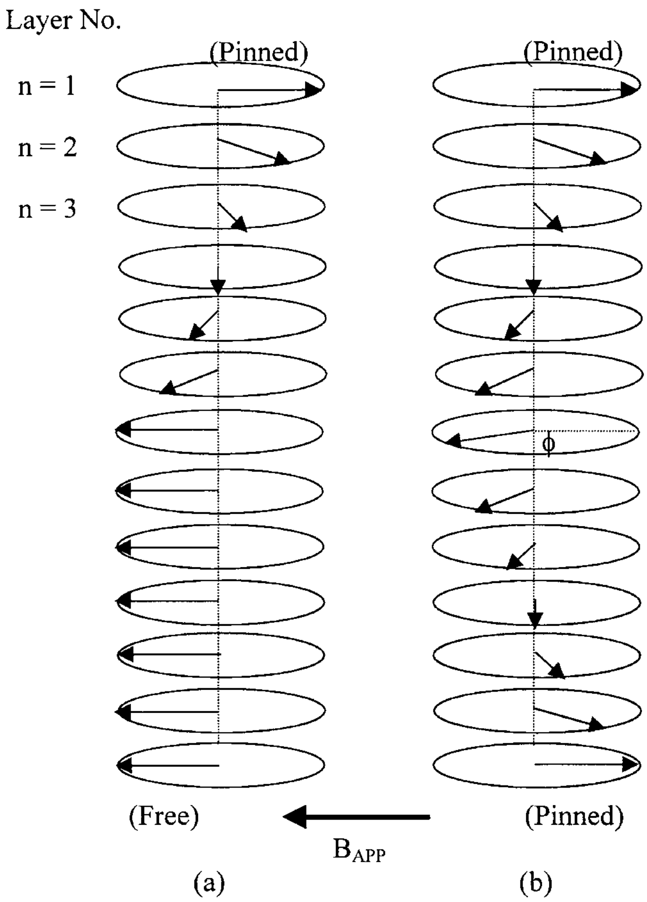


**Figure 1.** Schematic drawings of (a) a (0–180°) exchange-spring in a bilayer sample, (b) a symmetric exchange-spring (0–$\phi$–0) in a multilayer sample.

this time paying particular reference to the choice of boundary conditions. It is also shown that, despite the discrete nature of the soft ferromagnetic layer, Jacobi elliptic functions can still be used to advantage, even in the case of very short exchange-springs. In section 4, an analytical derivation of the result $B_B/B_{EX} = (\pi/2N)^2$ for a bilayer exchange-spring is given in terms of a small sinusoidal angular displacement. Finally, in section 5, the problem of exchange-spring penetration into neighbouring hard magnetic layers is examined, using numerical methods.

## 2. Numerical solution for exchange-springs

Schematic drawings of exchange-springs in a soft magnetic layer can be seen in figures 1(a) and (b). An exchange-spring (0–$\phi$), where $\phi$ can be 180°, is shown in figure 1(a). This spring is appropriate to a bilayer sample. In figure 1(b), a symmetric exchange-spring (0–$\phi$–0) can be seen. Such springs occur in multilayer samples. In both diagrams, it is assumed that the spins at the edges of the soft magnetic layer are pinned by the neighbouring hard magnetic layer, or layers, respectively.

Following Fullerton *et al* (1998), but with minor changes in notation, the total energy for an exchange-spring takes the form:

$$E = \sum_n \varepsilon(n) \tag{1}$$

where (i)

$$\varepsilon(n) = +\mu_{Fe} B_{APP} \cos\theta_n - \tfrac{1}{2}\mu_{Fe} B_{EX}[\cos(\theta_n - \theta_{n-1}) + \cos(\theta_{n+1} - \theta_n)] \tag{2}$$

(ii) the symbols possess their usual meanings and (iii) $B_{APP}$ is directed along the negative $z$-axis, as depicted in figure 1. Note that in writing equation (2), only nearest-neighbour interactions have been included and the anisotropy associated with the soft magnetic layer has been neglected.

For stability, the energy of the entire domain $E$ wall must be stable with respect to small partial variations in $\theta_n$:

$$\frac{\partial E}{\partial \theta_n} = 0. \tag{3}$$

In practice, the angle $\theta_n$ only appears explicitly in the energies of the $(n-1)$, $n$ and $(n+1)$th layers. So we can rewrite equation (3) in the form:

$$\frac{\partial E}{\partial \theta_n} = \frac{\partial E(\theta_n)}{\partial \theta_n} = 0 \tag{4}$$

where

$$E(\theta_n) = +\mu_{Fe} B_{APP} \cos\theta_n - \mu_{Fe} B_{EX}[\cos(\theta_n - \theta_{n-1}) + \cos(\theta_{n+1} - \theta_n)]. \tag{5}$$

Note however that equation (4) only holds when the complete set of equilibrium angles $\{\theta_n\}$ is known. Also care must be exercised with the spins at the beginning and end of the exchange-spring, pinned by the hard magnetic layers.

The equilibrium set of angles $\{\theta_n\}$ can be found, iteratively, in the following way. First, a trial set $\{\theta_n\}_0$ is chosen. This could be say a linear extrapolation from 0 to $+\pi$ to the middle of a symmetric exchange-spring, and $+\pi$ to 0 to the end. Initially therefore, the condition for stability (equation (4)) is almost certainly not met. Next, we define the initial gradient:

$$(E(\theta_n)')_0 = \left(\frac{\partial E}{\partial \theta_n}\right)_0 = -\mu_{Fe} B_{APP} \sin\theta_n + \mu_{Fe} B_{EX}[\sin(\theta_n - \theta_{n-1}) - \sin(\theta_{n+1} - \theta_n)]. \tag{6}$$

Our criterion therefore, for choosing the set $\{\theta_n\}$, is to minimize the gradients $E(\theta_n)'$ for all $n$.

This can be done in an iterative manner, which differs from the Monte Carlo method described by Fullerton *et al* (1998). Let us imagine that our initial set $\{\theta_n\}_0$ is fairly close to the equilibrium set, but that we can move even closer by setting

$$\left\{\begin{array}{c} \theta_{n-1} \to \theta_{n-1} + \partial\theta_{n-1} \\ \theta_n \to \theta_n + \partial\theta_n \\ \theta_{n+1} \to \theta_{n+1} + \partial\theta_{n+1} \end{array}\right\}. \tag{7}$$

Consequently, on substituting equations (7) into equation (6), expanding, and gathering only those terms which are linear in $\partial\theta$ we find

$$E(\theta_n)' \to (E(\theta_n)')_0 - E(\theta_n)\partial\theta_n - \mu_{Fe} B_{EX}[\cos(\theta_n - \theta_{n-1})\partial\theta_{n-1} + \cos(\theta_{n+1} - \theta_n)\partial\theta_{n+1}] = 0. \tag{8}$$

This equation can be re-expressed in the matrix form:

$$M\underline{\partial\theta} = \underline{(E(\theta_n)')_0} \tag{9}$$

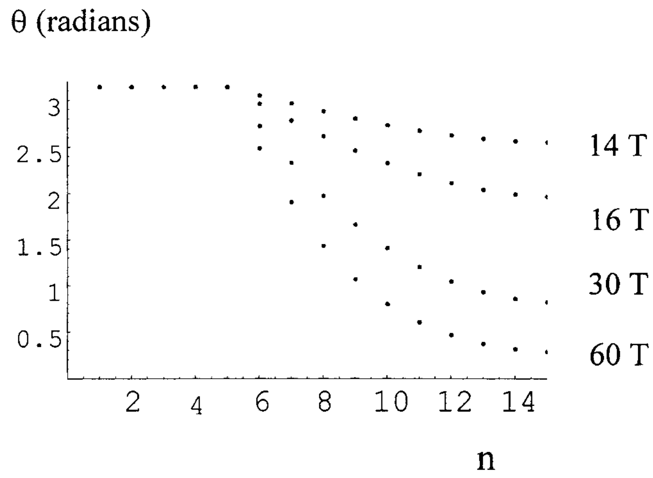


**Figure 2.** Computed shapes for a specific example of an 180° bilayer exchange-spring, for $N = 10$ monolayers, as a function of applied field.

where (i) $\underline{(E(\theta_n)')_0}$ and $\underline{\partial\theta}$ are column matrices, and (ii) $\underline{M}$ is an $N \times N$ matrix, whose elements can be easily deduced from equation (8). In practice, in the case of symmetric exchange-springs it is possible to exploit the symmetries inherent in the matrix $\underline{M}$, to reduce the calculation of the angles by a factor $\sim$2. Finally, on re-arranging equation (9) we find

$$\underline{\partial\theta} = \underline{M}^{-1}\underline{(E(\theta_n)')_0}. \tag{10}$$

So the new set of trial angles is given by

$$\underline{\theta} \to \underline{\theta} + \underline{\partial\theta}. \tag{11}$$

In practice, convergence is very rapid. Usually five iterations are more than enough, given a judiciously chosen initial set of angles.

Once the equilibrium set of angles $\{\theta_n\}$ has been determined, it is a straightforward matter to calculate the energy of the entire exchange-spring and to compare it with the energy of the simple ferromagnetic state (i.e. no exchange-spring). In practice, exchange-springs only form when the gain in Zeeman energy outweighs the concomitant loss in magnetic exchange energy. So the calculations presented in this paper are only valid for $T = 0$ K. Following Goto *et al* (1965), we shall denote the critical field required to create an exchange-spring as the bending field $B_B$.

In figure 2, the computed shapes for a 180° bilayer exchange-spring, with $N = 10$ monolayers, can be seen, as a function of applied field. From an examination of these curves, it is clear that for fields that just exceed the bending field $B_B$, the maximum deviation $\phi$ away from the ferromagnetic state $\theta_n = 0$ is very small. However, as the field is increased, more and more spins are pulled away from the anti-parallel alignment. We also note that while the change $\mathrm{d}\theta_n/\mathrm{d}n$ for the last spin in the chain is small it is not zero, except in the case of a very long exchange-spring where $\phi = 180°$ is reached long before the end of the chain. This point is taken up again in the next section.

In figure 3, the computed curves can be seen for a symmetric exchange-spring, with a nominal $N = 19$ spins. Given such curves, it is an easy matter to calculate the extra magnetic moment generated by the exchange-springs. This is illustrated in figure 4, which shows the reduced magnetization curves $\langle\cos\theta\rangle$ of the soft magnetic layer, as a function of both magnetic field and number $N$. It will be observed that the bending field $B_B$ increases sharply as the

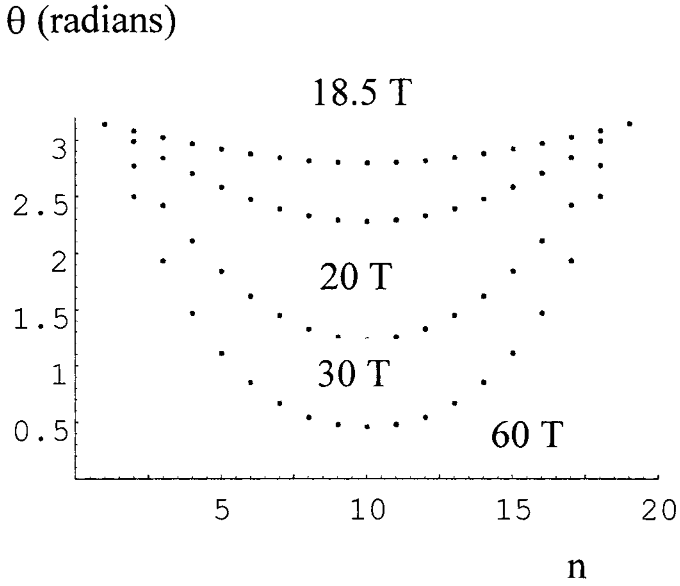


**Figure 3.** Computed shapes for an $N = 19$ monolayer symmetric exchange-spring in a multilayer sample, as a function of applied field.

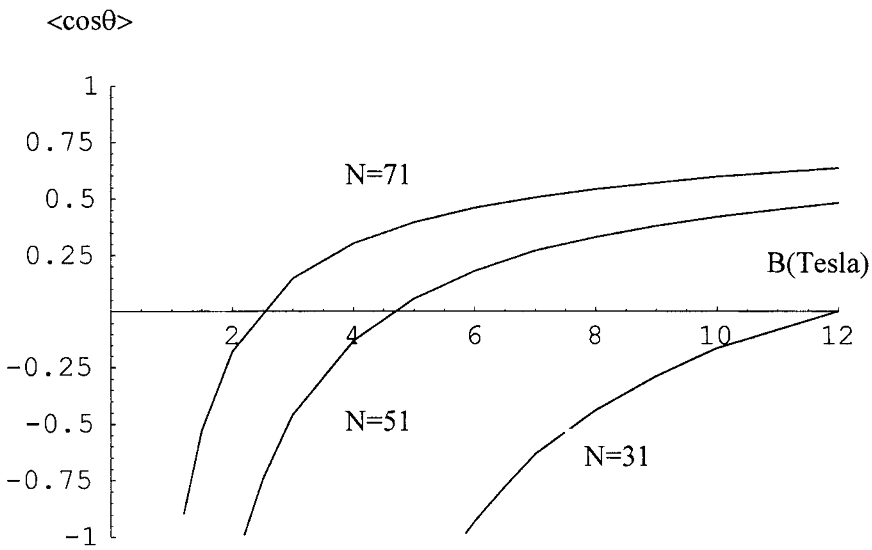


**Figure 4.** Reduced magnetization curves $\langle \cos\theta \rangle$ of the soft magnetic layer in a symmetric exchange-spring, as a function of magnetic field and number $N$.

number of spins $N$ in the soft layer is reduced. From computer calculations for a symmetric exchange-spring, we find that

$$\frac{B_B}{B_{EX}} = \left(\frac{\pi}{N}\right)^2 \tag{12}$$

to an accuracy of 1 in $10^4$. The reason for writing $\pi^2$ in equation (12), instead of a computed number, will become clear in section 4. Similarly, for a symmetric exchange-spring

$$\frac{B_B}{B_{EX}} = \left(\frac{\pi}{2N}\right)^2. \tag{13}$$

Note that the bending field for an equivalent number of spins $N$ is always lower in a bilayer exchange-spring, as expected.

From an examination of both equations (12) and (13), it is clear that the bending field $B_B$ is proportional to $1/w^2$ where $w$ is the width of the soft magnetic layer. As mentioned earlier, this result was obtained long ago by Goto *et al* (1965), for continuous bilayers. More recently, it has been shown that the inverse square law also applies for a soft magnetic sphere of radius $R$, embedded in a hard magnetic matrix (Skomski and Coey 1999). However, equations (12) and (13) represent the most transparent form of the relationship between the bending field $B_B$ and exchange field $B_{EX}$.

In section 4, an analytical derivation of equation (13) is presented and discussed. However, before leaving this section, it should be acknowledged that in obtaining the numerical results embodied in equations (12) and (13), there is a small problem about what to do with the end spins of the exchange-spring. For large number of spins this does not constitute a problem. But for small numbers of spins it does affect the exponent of $N$ and the constant appearing in equations (12) and (13). For example, consider the first spin in an exchange-spring, pinned by a hard magnetic layer. The question is: 'Does the pinned spin belong to the exchange-spring or the hard layer?'. In practice, we have found that consistent numerical results, for differing values of $N$, are obtained by ascribing an extra $\frac{1}{2}$ for each spin pinned by a hard layer.

In the next section, the analytic form of the exchange-spring in a bilayer system given by Goto *et al* (1965) is re-examined. These authors showed that the angular form of a continuous bilayer exchange-spring could be expressed in terms of Jacobi elliptic functions. However, their treatment only strictly holds true when the turn angle between neighbouring spins is very small. Thus for short discrete domain walls, numerical methods would appear to be mandatory. However, it will be shown that the analytical solution is surprisingly robust, even in the case of very short discrete exchange-springs.

## 3. Analytical expressions for exchange-springs

The spin configuration in question, is illustrated schematically in figure 1(a). This figure has been drawn to coincide with the magnetic spin configuration of Goto *et al* (1965). Note that the top layer ($n = 0$) is pinned by the presence of the hard magnetic layer.

As mentioned earlier, the partial derivative of the total energy $E$ with respect to $\theta_n$, keeping all other angles fixed, must be zero ($\partial E/\partial\theta_n = 0$). For a large number of monolayers $N$, the turn angle per layer will be very small. Consequently, it is permissible to expand $\theta_{n\pm1}$:

$$\theta_{n\pm1} = \theta_n \pm \frac{\mathrm{d}\theta_n}{\mathrm{d}n} \times 1 + \frac{1}{2}\frac{\mathrm{d}^2\theta_n}{\mathrm{d}n^2} \times (1)^2 \ldots. \tag{14}$$

To second order therefore, equation (4) takes the form

$$\frac{\partial E}{\partial\theta_n} = -\mu_{FE} B_{EX} \frac{\mathrm{d}^2\theta_n}{\mathrm{d}n^2} - \mu_{FE} B_{APP} \sin\theta_n = 0 \tag{15}$$

or alternatively

$$\frac{\mathrm{d}^2\theta_n}{\mathrm{d}n^2} + \alpha^2 \sin\theta_n = 0 \tag{16}$$

where (i) $B_{APP}$ is directed along the negative $z$-axis, as shown in figure 1(a), and (ii)

$$\alpha^2 = \left(\frac{B_{APP}}{B_{EX}}\right). \tag{17}$$

Note that if $N$ is large and the spacing $a$ between layers is small then

$$\frac{\mathrm{d}^2\theta_z}{\mathrm{d}z^2} + \beta^2 \sin\theta_z = 0 \tag{18}$$

where (i) we have multiplied equation (17) by $(1/a^2)$, and (ii) $\beta$ is given by

$$\beta^2 = \frac{1}{a^2}\left(\frac{B_{APP}}{B_{EX}}\right). \tag{19}$$

Equation (18) is essentially identical to that of Goto *et al* (1965), except that their constant $\beta^2$ is given by

$$\beta^2 = \frac{H_X M_s}{2A} \tag{20}$$

where (i) $H_X$ is the applied field, (ii) $M_s$ is the saturation magnetization, and (iii) $A$ is the exchange constant.

The solution to the differential of equation (18) is easily shown to be

$$\sin\tfrac{1}{2}\theta_z = k\,\mathrm{sn}[\beta z] \tag{21}$$

where sn is a Jacobi elliptic function of module $k$. For convenience we re-write equation (21) in the form

$$\sin\tfrac{1}{2}\theta_z = k\,\mathrm{sn}\left[K\frac{z}{d}\right] \tag{22}$$

where the first quarter period $K$ of the sn function is given by

$$K = \left(\frac{d}{a}\right)\sqrt{\frac{B_{APP}}{B_{EX}}} = N\sqrt{\frac{B_{APP}}{B_{EX}}}. \tag{23}$$

Note that the first quarter period $K$ and the module $k$ are related via the elliptic integral:

$$K = \int_0^{\pi/2} \frac{\mathrm{d}\theta}{\sqrt{1 - k^2\sin^2\theta}}. \tag{24}$$

Thus either $K$ or $k$ is sufficient to define the Jacobi elliptic function sn in equation (22).

At this point in the discussion, it is necessary to differentiate between the 180° bilayer and symmetric multilayer exchange-springs. For a bilayer exchange spring, we find, on substituting equation (13) into equation (23),

$$K = \left(\frac{\pi}{2}\right)\sqrt{\frac{B_{APP}}{B_B}}. \tag{25}$$

This result is in agreement with Goto *et al* (1965) (their equation (2.5)), but obtained via a different route. Note that $K$ is necessarily greater than $(\pi/2)$, otherwise the stable spin configuration reverts to that of the ferromagnetic state.

The solution defined by equations (22) and (25) possesses the boundary conditions

$$\begin{aligned} \theta(z) &= 0^\circ && \text{at } z = 0 \\ \frac{\mathrm{d}\theta(z)}{\mathrm{d}z} &= 0 && \text{at } z = d \end{aligned} \tag{26}$$

**Table 1.** Some calculated values of $K$ for a bilayer spring of $N = 10$ spins. For an exchange field $B_{EX}$ of 600 T, the calculated value of $B_B$ was found to be 13.396 T.

| $B_{APP}$ | $K$ (calc, from $B_B$) | $\theta_N$ (last angle (rad)) | $K$ (calc, from $\theta_N$) |
|---|---|---|---|
| 14 | 1.6059 | 2.545 078 | 1.6065 |
| 15 | 1.6638 | 2.193 847 | 1.6622 |
| 16 | 1.7168 | 1.962 434 | 1.7193 |
| 30 | 2.3508 | 0.817 358 | 2.3663 |
| 60 | 3.3245 | 0.282 0338 | 3.3602 |

**Table 2.** Some calculated values of $K$ for a symmetric exchange-spring of $N = 17$ monolayers. The value of $B_B$ was found to be 18.231 T, for $B_{EX} = 600$ T.

| $B_{APP}$ | $K$ (calc, from $B_B$) | $\theta_N$ (last angle (rad)) | $K$ (calc, from $\theta_N$) |
|---|---|---|---|
| 18.5 | 1.5823 | 2.794 319 | 1.5827 |
| 20 | 1.6452 | 2.276 331 | 1.6476 |
| 30 | 2.0150 | 1.229 665 | 2.0298 |
| 60 | 2.8496 | 0.456 872 | 2.8961 |

(see Goto *et al* 1965, equation (2.3)). The first of these conditions simply states that the spins are pinned at $z = 0$. However the second condition is dictated by the properties of the sn Jacobi function at $u = K$, the first quarter period. Here the gradient of the continuous sn function is necessarily zero, by virtue of the periodic nature of the sn function. In practice however, discrete calculations reveal that while the gradient $\mathrm{d}\theta_n/\mathrm{d}n$ for the last spin is not necessarily zero, the use of Jacobi sn functions still provides a very good description of a bilayer exchange-spring. This is illustrated in table 1, where we have listed some of the properties of an exchange-spring with only $N = 10$ monolayers. It will be observed that the values of $K$, calculated via two different routes, are in very good agreement with each other. This is perhaps surprising, given the large turn angles between individual spins in large applied fields.

The situation for symmetric exchange-springs, with an odd number of spins, is slightly different. Here the gradient $\mathrm{d}\theta_n/\mathrm{d}n$ of the middle spin is necessarily equal to zero, and so the periodicity of the Jacobi sn function is upheld, even in the case of very short exchange-springs. Thus even better agreement between the predictions of the Jacobi elliptic function and that of direct calculation might be anticipated. On substituting equation (12) into (23) we find

$$K_s = \pi\sqrt{\frac{B_{APP}}{B_B}} \tag{27}$$

for a symmetric exchange-spring. But note that when $z = d/2$, the angle $\theta_M$ at the centre of the spring is given by

$$\sin\frac{1}{2}\theta_M = k\mathrm{sn}\left[\frac{1}{2}K_s\right] = k\mathrm{sn}\left(\frac{\pi}{2}\sqrt{\frac{B_{APP}}{B_B}}\right). \tag{28}$$

This equation has precisely the same form as that for the bilayer problem, embodied in equations (22) and (25), except, of course, that the bending fields $B_B$ differ in the two cases. This suggests therefore that we should use equations (22) and (25), for both problems, but with the proviso that (i) in the case of the bilayer problem $z$ terminates at $d$, whereas in the symmetric exchange-spring $z$ terminates at 2 $d$, and (ii) the bending fields are different. Some selected results for $N$ set nominally equal to 17 monolayers can be seen in table 2.

As with the bilayer results, it will be seen that the agreement between the calculated results and the predictions of the elliptic Jacobi function is excellent, despite the discrete nature of the exchange-spring.

## 4. The bending field transition

The numerical calculations presented in section 2 show that when $B_{APP}$ just exceeds the bending field $B_B$, the angles made by the exchange-spring are very close to $\theta = 0$. This suggests, therefore, that it will be advantageous to study the stability of the ferromagnetic state, with respect to a small collective instability.

To make further progress, it is necessary to specify a form for the collective angular displacement. For the bilayer exchange-spring, we shall assume that the spins are subjected to a sinusoidal standing wave ($\lambda = 4d$) of amplitude $\phi_0$ on the ferromagnetic structure, which we write in the form:

$$\theta_n = \phi_0 \sin\left(\frac{\pi z}{2d}\right) = \phi_0 \sin\left(\frac{\pi n}{2N}\right). \tag{29}$$

Here we have assumed (i) that the amplitude $\phi_0$ of the sinusoidal disturbance is very small, (ii) the number of monolayers in the magnetically soft layer is $N$ and (iii) the total width of the soft magnetic layer is given by $d = Na$.

The following approximations are useful:

$$\begin{aligned} \cos(\theta_n - \theta_{n-1}) &\approx \left[1 - \frac{1}{2}\left\{\phi_0\left(\frac{\pi}{2N}\right)\cos\left(\frac{\pi n}{2N}\right)\right\}^2 \cdots\right] \\ \cos(\theta_{n+1} - \theta_n) &\approx \left[1 - \frac{1}{2}\left\{\phi_0\left(\frac{\pi}{2N}\right)\cos\left(\frac{\pi n}{2N}\right)\right\}^2 \cdots\right]. \end{aligned} \tag{30}$$

These can be used to show that the energy $\varepsilon(n)$ of equation (2) can be re-written:

$$\varepsilon(n) = \varepsilon_F(n) - \frac{1}{2}\mu_{Fe}\phi_0^2\left[B_{APP}\sin^2\left(\frac{\pi n}{2N}\right) - \left(\frac{\pi}{2N}\right)^2 B_{EX}\cos^2\left(\frac{\pi n}{2N}\right)\right] \tag{31}$$

which is to be compared with the ferromagnetic state:

$$\varepsilon_F(n) = \mu_{Fe}B_{APP} - \mu_{Fe}B_{EX}. \tag{32}$$

From a comparison of equations (31) and (32), it is clear that the exchange fan will form only if the gain in Zeeman energy offsets the concomitant loss in magnetic exchange. This will occur if

$$B_{APP}\sum_n \sin^2\left(\frac{\pi n}{2N}\right) \geqslant \left(\frac{\pi}{2N}\right)^2 B_{EX}\sum_n \cos^2\left(\frac{\pi n}{2N}\right) \tag{33}$$

which reduces to that of equation (13). Thus we have provided an explanation for the $(\pi/2N)^2$ behaviour, obtained earlier by direct numerical calculation. A similar argument can be advanced for a symmetric exchange-spring.

In passing, we note that the formation of an exchange-spring can be viewed as an elementary excitation of the soft magnetic layer that goes 'soft', as the applied field $B_{APP}$ approaches the critical bending field $B_B$. However, this conclusion only strictly applies when $T = 0$ K.

Finally, we give a rather belated justification for the choice of a sinusoidal distortion. We have already shown that the solution for an exchange-spring is given by equation (22). Since this equation holds for all $B_{APP}$ in excess of $B_B$, this suggests that our earlier discussion should

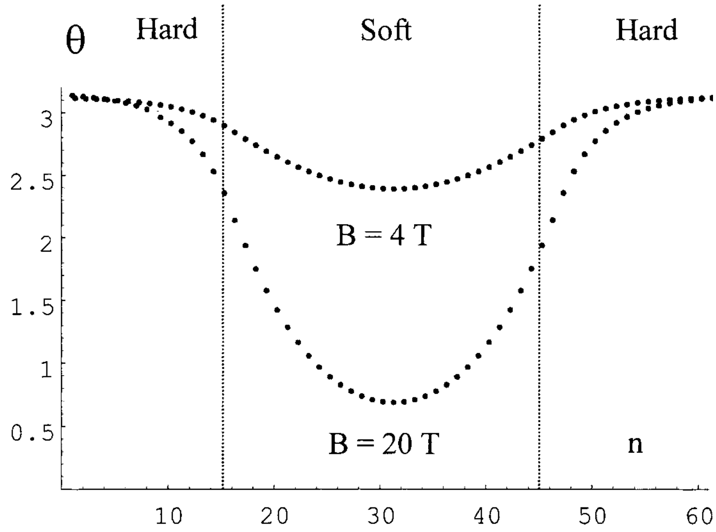


**Figure 5.** Computed shapes for an $N = 31$ monolayer symmetric exchange-spring, showing penetration into the hard magnetic layers on either side, in a multilayer sample.

have been couched in terms of Jacobi elliptic functions. However, we shall now show that for small deviations away from the ferromagnetic state, the Jacobi sn function reduces to a simple sine function.

If the maximum deviation $\phi$ away from the ferromagnetic state is small, it follows from equations (24) and (25) that $k$ must be small. Thus on expanding equation (24) we find:

$$K \approx \frac{\pi}{2}\left(1 + \left(\frac{k}{2}\right)^2 + \cdots\right) \tag{34}$$

to second order in $k$. Note that in the limit $k \to 0$, $K \to \pi/2$.

Next we observe that for elliptic functions:

$$u = \int_0^{\phi} \frac{\mathrm{d}\theta}{\sqrt{1 - k^2 \sin^2\theta}} \tag{35}$$

where the amplitude of $u$ (am[$u$]) is $\phi$ and

$$\mathrm{sn}[u] = \sin[\phi]. \tag{36}$$

On expanding equation (35) therefore we find

$$u = \left(\phi + \left(\frac{k}{2}\right)^2\left[\phi - \frac{1}{2}\sin\phi\right]\right) \tag{37}$$

to second order in $k$. Consequently, as $k \to 0$, $u \to \phi$, and so

$$\mathrm{sn}[u] \to \sin[u]. \tag{38}$$

We are now in a position to prove our original assumption. Returning to the Jacobi elliptic solution of equation (22), and making use of equations (34) and (38), we find that

$$\theta_z = k\,\mathrm{sn}\left[K\frac{z}{d}\right] \approx k\sin\left[\frac{\pi}{2}\left(\frac{z}{d}\right)\right] \tag{39}$$

in the limit of small $k$. Consequently, on comparing equations (29) and (40), we see that the initial angular deviation away from the ferromagnetic state is indeed sinusoidal, with amplitude $\phi_0 = k$.

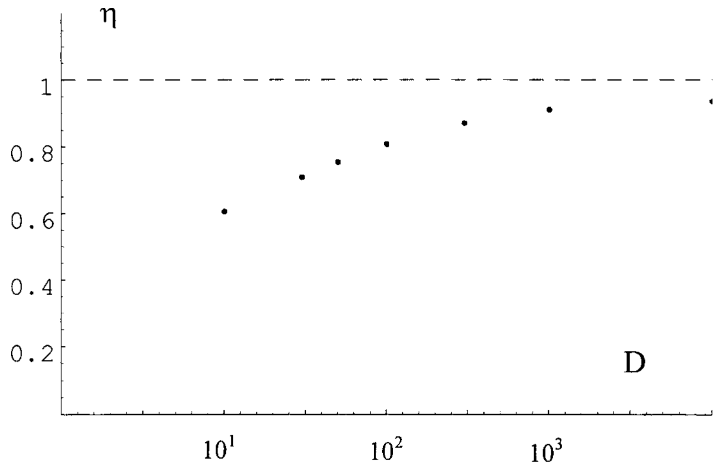


**Figure 6.** Calculated bending fields $B_B$, in reduced form $\eta$, as a function of the axial anisotropy parameter $D$ (see text). The calculations are for the same exchange-spring as described in figure 5.

## 5. Exchange penetration into the magnetically hard layers

In all of the work presented so far, it has been assumed that the spins in a soft magnetic layer are pinned ($\theta = 0$) at the hard magnetic boundaries. In practice however, some penetration of the exchange spring into the magnetically hard layer is likely to occur. This situation is easily catered for, by a modest adaptation of equations (1) to (11) to include the anisotropy of the spins in the hard layer. Such calculations allow one to determine both the shape and depth of penetration of exchange springs into the hard magnetic layers. However, as expected equations (12) and (13) no longer hold, except in the case of very high magnetic anisotropy. An example of a symmetric exchange spring containing $N = 31$ monolayers, sandwiched between two hard magnetic layers (15 either side shown), can be seen in figure 5. These results have been modelled on the MBE grown $DyFe_2/YFe_2$ films recently discussed by Sawicki *et al* (2000). The magnetic anisotropy associated with the $Dy^{3+}$ ions has been simulated with a simple axial term:

$$E_A = D\cos(2\theta) \tag{40}$$

where $D$ has been set equal to 10 K. This, perhaps rather low value of $D$ has been chosen because it allows considerable penetration of the exchange spring into the hard $DyFe_2$ layers to take place. This has the effect of reducing the bending field $B_B$ required to set up an exchange spring, because $N$ has effectively been increased. We find that $B_B$ is 3.7308 T, for an exchange field $B_{EX} = 600$ T. This figure is much lower than the value of 6.1621 T obtained using equation (12). To examine this feature further we have plotted the reduced bending field:

$$\eta = B_B(D)/B_B(D \to \infty) \tag{41}$$

as a function of the axial anisotropy parameter $D$. It will be observed that very large values of the parameter $D$ are required, before the bending field $B_B$ approaches the value given by equation (12). In practice therefore, the use of equations (12) and (13) will almost certainly give rise to overestimates for the bending field $B_B$.

Finally, we note that Skomski and Coey (1999) have also discussed the problem of exchange-spring penetration into hard magnetic layers. In particular, they have given an eigenvalue equation that can be used to determine the bending field (their $H_N$), using numerical methods.

## 6. Conclusions

The properties of soft magnetic exchange-springs in both bilayer and multilayer samples, consisting of alternatively hard and magnetically soft layers, have been investigated using both numerical and analytical methods. It has been shown that despite the rather complicated shape of the exchange-springs, a very simple relationship exists between the bending field $B_B$, the exchange field $B_{EX}$, and the number of layers $N$ in the soft magnetic layer. In the mean field model, $B_B/B_{EX} = (\pi/2N)^2$ for a bilayer spring, whereas for multilayer exchange-springs $B_B/B_{EX} = (\pi/N)^2$. In addition, it has been shown that the Jacobi elliptic sn function, originally developed for continuous 180° bilayer exchange-springs, provides a robust description of both discrete bilayer and symmetric exchange-springs, even when very few monolayers $N$ are involved. Finally, the problem of soft exchange spring penetration into neighbouring hard magnetic layers has been investigated. Calculations show that this is an important effect, which can reduce the bending field transition by some 10–50%, depending on the hardness of the hard magnetic layers.

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